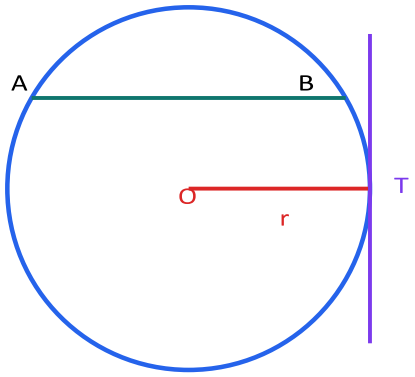
11 Circle Theorems

Circle Radius Tangent Chord/Secant

Introducing Terminology

Key circle parts and lines.

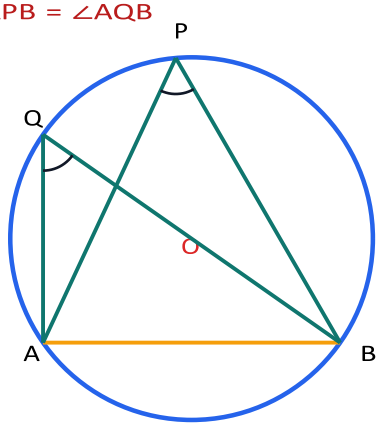


Practice: Identify a radius and a tangent in the diagram.
Answer: Radius: OT. Tangent: the purple line at T.

1. Same-arc angles

Angles standing on the same chord/arc are equal.

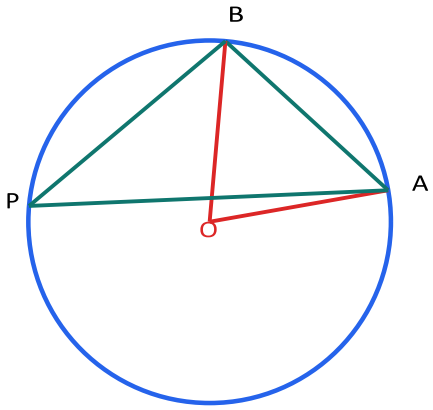
$\angle APB = \angle AQB$



Practice: If $\angle APB = 38°$, find $\angle AQB$. Answer: $38°$.

2. Center angle is twice

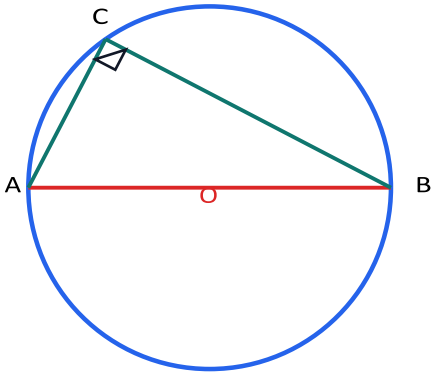
$\angle AOB = 2\angle APB$ on the same arc AB.



Practice: If $\angle APB = 27°$, what is $\angle AOB$? Answer: $54°$.

3. Semicircle gives 90°

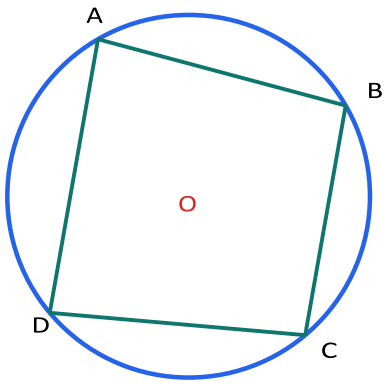
Diameter subtends a right angle at the circumference.



Practice: If AB is a diameter and $\angle ABC = 35°$, find $\angle BAC$. Answer: $55°$.

4. Cyclic quadrilateral

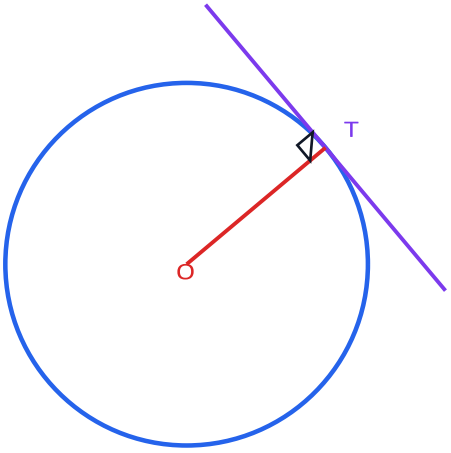
Opposite angles sum to 180°.



Practice: In cyclic ABCD, if ∠A = 72°, find ∠C.
Answer: 108°.

5. Tangent ⊥ Radius

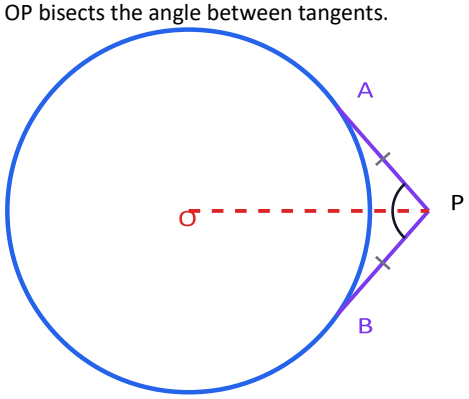
Tangent ⊥ radius at the point of contact.



Practice: What is the angle between OT and the tangent at T?
Answer: 90°.

6. Two tangents from one point

PA = PB; OP bisects the angle between tangents.

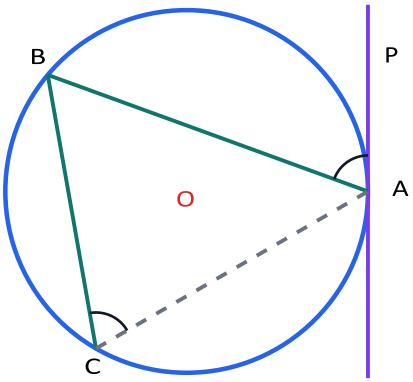


Practice: If PA = 12, find PB. Answer: 12.

7. Alternate segment theorem

Angle (tangent, chord) = angle in opposite segment.

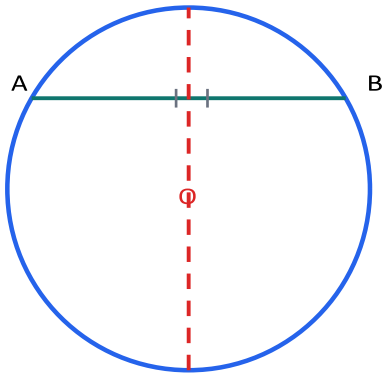
∠PAB = ∠ACB



Practice: If ∠PAB = 42°, find ∠ACB. Answer: 42°.

8. Chord bisector theorem

The perpendicular bisector of a chord passes through O.

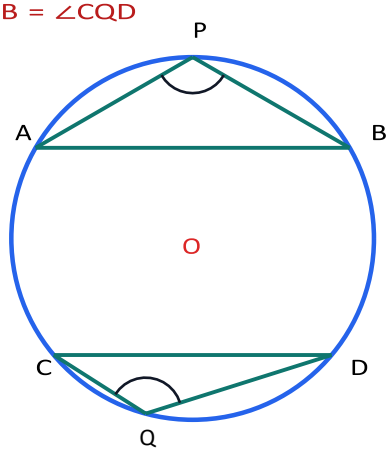


Practice: A line bisects chord AB at right angles. Where does it pass?
Answer: Through the center O.

9. Equal chords ⇒ equal angles

Equal chords subtend equal angles at the circumference.

$\angle APB = \angle CQD$



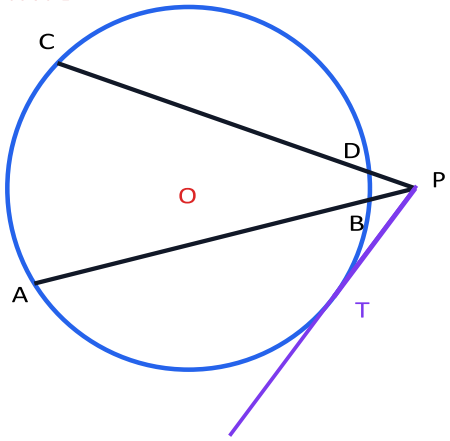
Practice: If ∠APB = 149°, find ∠CQD. Answer: 149°.

10. Two secant theorem

From external point P: PA·PB = PC·PD; and $PT^2 = PA \cdot PB$ (tangent–secant).

$PA \cdot PB = PC \cdot PD$

$PT^2 = PA \cdot PB$

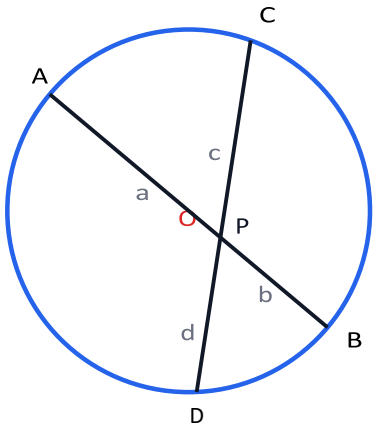


Practice: If PA = 12 and PB = 3, find PT. Answer: 6.

11. Chord–chord power

Intersecting chords: (AP)(PB) = (CP)(PD).

$a \cdot b = c \cdot d$



Practice: If a = 4, b = 9, c = 6, find d. Answer: 6.